

OPTIMAL DESIGN OF REINFORCED CONCRETE T-BEAM FLOORS

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ABSTRACT

Mathematical programming techniques have been used to minimize the cost of reinforced concrete T-beam floor. The floor system consists of one way continuous slab and simply supported T-beams. The study presents a formulation based on elastic analysis followed by the ultimate strength method of design with the consideration of serviceability constraints as per ACI Code. The formulation of optimization problem has been made by utilizing the interior penalty function method as an optimization method with the purpose of minimizing the objective function representing the cost of one-meter length of the floor system. The cost includes cost of concrete, reinforcement, and formwork. The design variables considered in this study are the dimensions and the amounts of reinforcement for the slab and beams, in addition to the spacing of the beams. Many examples are solved to show the effect of these design variables on the optimum solution of the floor system. The effect on the optimum design of the compressive strength of concrete, yield strength of steel, concrete cost ratios, and formwork cost ratios has also been studied.

INTRODUCTION:

Most of structural design problems have several solutions, and the aim of the designer is to choose the best possible or optimum solution. Optimization, in its broadest sense, can be applied to solve engineering problems. Design of reinforced concrete (RC) structures is one of several applications of optimization in the field of structural design.

The optimum design of RC T-beam has been studied by many researchers. Chou⁽¹⁾ used the Lagrange multipliers as an optimization method in the optimum design of RC T-beam sections. An objective function was evaluated by summing the cost of the reinforcement and concrete. The cost of formwork and dead weight of the beam were neglected. An algorithm to obtain the optimum design of the T-beam was presented by Balaguru⁽²⁾. Dead weight of the beam was considered as a variable. Cost of concrete, steel, and formwork were considered. Subramanyam and Adidam⁽³⁾ used the limit state method and mathematical programming to get optimal design of a typical T-beam floor conforming to Indian standard IS: 456- 1978 and British standard CP110- 1972. Different cost ratios for materials and formwork were considered. The effect of using different grades of concrete on the optimal design was examined.

The present paper deals with the design of RC T-beam floor in accordance with ACI- code specifications⁽⁴⁾ using the ultimate load theory. It is intended to incorporate a mathematical programming method in order to minimize the cost of T-beam floor. The spacing of the beams is

treated as one of the design variables. The dead weight of the slab and the beams is also considered as a variable.

FORMULATION OF THE PROBLEM METHOD OF ANALYSIS

T-beam floor is one of the most common structural forms in the field of RC floor system. Therefore, any saving that can be made will be highly beneficial. In the present study a typical T-beam floor, shown in (Fig.1), is optimally designed for minimum cost. The slab is designed as one way continuous slab based on the moment coefficients given by the ACI 318- 89 code. The span length used in the design procedure of the floor system is taken equal to the center to center distance between beams.

The beam is designed as a singly reinforced, simply supported T-beam. Figures (2) and (3) show the details of slab and beams reinforcement. Two types of loading are considered, these are: 1) Live load 2) Dead load.

Internal design forces (moments, and shears) are obtained for the combined effect of the dead and live loads.

DESIGN PARAMETERS

In the design procedure of the RC T-beam floor, some parameters are considered to be constant along the design processes, and they should be given at the start of the program. These include:

1. The live load on the slab.
2. The total length and width of the slab.

3. The minimum cover for the reinforcement of the beam and the slab.
4. The compressive strength of the concrete and the yield stress of steel.
5. The ratio of the cost of concrete per cubic meter to cost of reinforcement per newton, and the ratio of the cost of formwork per square meter to cost of reinforcement per newton.

The other parameters are considered as design variables and some of which belong to the slab including:

1. The thickness of slab.
2. The area of tension reinforcement per unit width at the top of the slab at the first interior support.
3. The area of tension reinforcement per unit width at the top and bottom of the slab for the end spans.
4. The area of tension reinforcement per unit width at the top and bottom of the slab for intermediate spans.

Other design variables belong to the beams and may be summarized as:

- 1) The center to center distance between beams, span length of slab.
- 2) The area of tension reinforcement for the beam.
- 3) The total depth of the T- beam.

CONSTRAINTS

The design of RC T- beam floor is required to satisfy two groups of requirements, namely, ultimate strength requirements and serviceability requirement of deflection in accordance with ACI code^[4].

Constraints for slab

Flexural constraints

For the critical sections of slab, the moments of resistance per unit width of slab should not be less than the bending moments per unit width of slab due to factored loads under the worst combination of loads.

$$mus_A \leq m_A \quad (1)$$

$$mus_B \leq m_B \quad (2)$$

$$mus_C \leq m_C \quad (3)$$

$$mus_D \leq m_D \quad (4)$$

where mus_A , mus_B , mus_C , and mus_D are the ultimate bending moments per unit width of slab. m_A , m_B , m_C , and m_D are the moments of resistance per unit width of slab. A, B, C and D are the critical sections of slab shown in Fig.(2).

The section moment capacity can be calculated as,^[4]

$$m_i = \phi_1 \cdot A_{ssi} \cdot f_y \cdot d_s^2 \cdot \left[1 - \frac{A_{ssi} \cdot f_y}{1.7 f_c' \cdot b_s \cdot d_s} \right] \quad (5)$$

where:

i = the critical sections A, B, C, or D

Φ_1 = moment reduction factor = 0.9

A_{ssi} = tensile reinforcement required at the critical section (i) of slab.

f_y = yield stress of the steel

f_c' = cylinder compressive strength of concrete.

d_s = effective depth of slab = $h_f - d_2$

h_f = thickness of slab

d_2' = the distance from the centroid of the tension steel area to the tension face of the slab,

b_s = one-meter width of slab.

A limiting constraint is also employed to ensure that the tension reinforcement of slab at the critical sections are not less than the area of shrinkage and temperature reinforcement. Thus;

$$A_{s_{min}} \leq A_{ssi} \quad (6)$$

(4)

$A_{ss_{min}}$ = the area of shrinkage and temperature reinforcement per unit length of slab.

The area of shrinkage reinforcement for the slab is given by:

$$A_{ss_{min}} = \rho_{sh} \times h_f \times b_s$$

where: ρ_{sh} = steel ratio for shrinkage^[4].

Deflection Constraints

In order to limit the deflection to permissible limits, the ratio of span length (l_s) to the effective depth (d_s) of the slab should not exceed, μ , the value given by ACI- code.

$$\frac{l_s}{d_s} \leq \mu \quad (7)$$

Shear Constraints

The effective depth of the slab, which is limited by deflection constraint, should also satisfy the shear force constraint. Thus:

$$F_{US} \leq \Phi V_{CS} \quad (8)$$

where:

F_{US} = ultimate shear force in the slab

$$= 1.15 \frac{W_{us} l_s}{2}$$

W_{US} = applied factored distributed load on the slab

l_s = span length (C/C distance between beams)

V_{CS} = shear strength provided by concrete cross- section of the slab

$$= \frac{1}{6} \Phi_2 \sqrt{f_c'} b_s d_s$$

Φ_2 = shear reduction factor = 0.85

Constraints for Beams

Flexural Constraints

The constraint considered for the critical section of the beam is that the moment of resistance of the beam should not be less than the maximum bending moment due to ultimate loads. Thus;

$$M_{ub} \leq M \quad (9)$$

where:

M_{ub} = maximum bending moment on the beam.

M = the ultimate moment of resistance of the cross-section of the beam.

Since the T-beam is designed as a rectangular beam, the moment capacity of the cross-section for the beam can be calculated as,

$$M = \phi_1 \times A_{sb} \times f_y \times d_b^2 \times \left[1 - \frac{A_{sb} \times f_y}{1.7 \times f_c' \times B \times d_b} \right]$$

where:

A_{sb} = area of tension reinforcement of the beam.

d_b = effective depth of the beam. = $H - d_1$

H = total depth of the beam.

d_1 = the distance from the centroid of tension steel area to the tension face of the beam.

B = effective width of the flange which is the smaller of:

- 1) $16 h_f + b_w$
- 2) l_s , or
- 3) $1/4 L_b$

L_b = total length of the beam.

Reinforcement limitations are also considered in the flexural design of the beam: these are

$$1 - \rho \leq \rho_{max} \quad (10)$$

where:

$$\rho = \text{steel ratio of the beam} = \frac{A_{sb}}{b_w d_b}$$

ρ_{max} = maximum steel ratio permitted by ACI - code^[4]

$$= 0.75 \times \left[0.85 \times \beta_1 \cdot \frac{f_c'}{f_y} \cdot \frac{600}{600 + f_y} \right]$$

2- Minimum reinforcement of the beam is limited by:

$$\rho_{min} \leq \rho \quad (11)$$

where:

$$\rho_{min} = \text{minimum steel ratio required} \\ = 1.4 / f_y$$

Shear Design

For simply supported beam, the shear force distribution is as shown in Fig.(4). Therefore, half of beam span is divided into three regions and the maximum shear force for each region controls the shear design.

The shear resistance of the cross-section for the beam should not be less than the shear force due to the applied factored load, at that cross-section. Thus

$$V_u \leq \phi_2 V_n \quad (12)$$

where:

V_u = the applied shear force

V_n = the shear resistance of the cross-section = $V_{cb} + V_s$

V_{cb} = the nominal shear strength provided by the concrete = $\frac{1}{6} \sqrt{f_c'} b_w d_b$

V_s = the nominal shear strength provided by the shear reinforcement.

$$= A_{sv} \cdot f_y \cdot d_b / s$$

A_{sv} = area of shear reinforcement within a distance (two legs).

s = spacing of shear reinforcement.

In addition to satisfying the shear design requirements the following limits are considered.

1. All sections where (V_u) is less than ($\phi_2 \cdot V_c$) are designed for the minimum shear reinforcement as follows

$$A_{sv \min} = \frac{b_w \times s}{3 \times f_y}, \text{ or}$$

$$s_{\max} = \frac{3 \times A_{sv} \times f_y}{b_w}$$

(13)

This will give an upper limit for the spacing of stirrups.

2. Maximum spacing of stirrups is limited as follows:

$$s \leq \frac{d}{2} \text{ or } 600 \text{ mm if } V_s \leq \frac{1}{3} \sqrt{f_c'} b_w d_b \quad (14)$$

and

$$s \leq \frac{d}{4} \text{ or } 300 \text{ mm if } V_s > \frac{1}{3} \sqrt{f_c'} b_w d_b \quad (15)$$

From practical consideration, the minimum spacing of stirrup is taken as 100 mm.

Another constraint is also considered in the shear design of the beam, as

$$V_s \leq \frac{2}{3} \sqrt{f_c'} b_w d_b \quad (16)$$

Deflection Constraint

Deflection constraint limits the deflection of the beam to permissible limits by

$$\frac{L_b}{d_b} \leq \mu_1 \quad (17)$$

where:

μ_1 = value given by ACI-code.

Objective Function

The objective function is the cost of all floor system per meter length. The total cost, C , consists of the cost of slab and the cost of beams. They are calculated as follows.

For the slab

Cost of concrete = $C_{cs} = h_f \cdot L \cdot R_1$

Cost of reinforcement = C_{rs}

$$= \{A_{st} + A_{sb}\} \gamma_s$$

Cost of formwork = $C_{fs} = L \cdot R_2$

For the beam

Cost of concrete = $C_{cb} = (Hh_f) bw \cdot N \cdot R_1$

Cost of reinforcement = $C_r =$

$= N \cdot [A_{sb} + A_v] \cdot \gamma_s$

Cost of formwork = C_{fb}

$= 2N(H-h_f)R_2$

where:

R_1 = the ratio of cost of concrete per cubic meter to cost of reinforcement per newton.

A_v = volume of stirrups in one meter length of the beam.

R_2 = the ratio of cost of formwork per square meter to cost of reinforcement per newton.

γ_s = specific weight of steel.

A_{st} = total reinforcement of slab.

N = number of beams for the floor system.

L = length of slab

Thus, C may be expressed mathematically as:

$$C = [C_{cs} + C_{cb}] + [C_{rs} + C_{rb}] + [C_{fs} + C_{fb}] \quad (18)$$

Solution Procedure

The optimization problem formulated in the previous sections is a constrained non-linear programming problem. Such problem can be solved by the interior penalty function method using the sequential unconstrained minimization technique⁽⁵⁾. Davidon-Fletcher-Powell algorithm is employed to find the search direction and cubic interpolation method to trap the minimum nondimensional search.

In the present problem, some constraints have maximum allowable value much larger than that of other constraints. This has bad effect on the convergence rate during the minimization of penalty function. Thus, it is decided to normalize the constraints so that they vary between 1 and 0 as far as possible. For the constraints of the

present problem, the normalization can be done as follows:

$$\frac{m_{usA}}{m_A} - 1 \leq 0 \quad (19)$$

$$\frac{m_{usB}}{m_B} - 1 \leq 0 \quad (20)$$

$$\frac{m_{usc}}{m_c} - 1 \leq 0 \quad (21)$$

$$\frac{m_{usD}}{m_D} - 1 \leq 0 \quad (22)$$

$$\frac{A_{ssmin}}{A_{ssi}} - 1 \leq 0 \quad (23)$$

$$\frac{I_s}{ds \times \mu} - 1 \leq 0 \quad (24)$$

$$\frac{F_{us}}{\phi_2 \cdot V_{cs}} - 1 \leq 0 \quad (25)$$

$$\frac{M_{ub}}{M} - 1 \leq 0 \quad (26)$$

$$\frac{\rho}{\rho_{max}} - 1 \leq 0 \quad (27)$$

$$\frac{\rho_{min}}{\rho} - 1 \leq 0 \quad (28)$$

$$\frac{V_s}{\frac{2}{3} \times \sqrt{f_c'} \times bw \times d_b} - 1 \leq 0 \quad (29)$$

$$\frac{L_b}{d_b \times \mu_1} - 1 \leq 0 \quad (30)$$

Applications And Results

A computer program for the optimum design of RC T-beam floor has been written to embody the given formulation. The program has been used to deal with different applications. The solved problems are:

1. The optimum design of the floor system is carried out for different cases of design variables. Each case has been treated twice; the first with and the second without considering the

formwork.

2. A parametric study has been made to show the influence of the compressive strength of concrete (f_c') and yield stress of steel (f_y) on the optimum design.

3. The effect of the variation of cost ratio on the optimum design has been studied.

The following general data are used:

Live load = $w_L = 4 \text{ kN/m}^2$

Slab length = $L = 12 \text{ m}$

Beam length = $L_b = 8 \text{ m}$

Rib width = $b_w = 200 \text{ mm}$

$f_y = 400 \text{ N/mm}^2$

$f_c = 20 \text{ N/mm}^2$

$d_1 = 60 \text{ mm}$

$d_2 = 20 \text{ mm}$

THE CONVENTIONAL DESIGN

The conventional design is done in accordance with ACI code using the general given data. The distance between centers of beams is taken equal to (3 m).

a) Slab design

The slab thickness (assumed) = 150 mm

Area of tension reinforcement at critical sections A, B, C and D = 270 mm^2

Shrinkage reinforcement = 270 mm^2

b) Beam Design

Total depth (assumed) = 700 mm

Area of long. reinforcement = 1530 mm^2

Shear reinforcement is 10 mm closed stirrups at 300 mm C/C for whole span of the beam.

OPTIMUM DESIGN

Example (1)

In this example the floor system is designed with two design variables, the total depth and tensile reinforcement of the beam. The results are illustrated in Table(1)

Example (2)

In this case the optimum design of RC T-beam floor is done with three design variables; the thickness of slab, total depth and tensile reinforcement of the beam. The distance (l_s) is treated in two ways. In the first, (l_s) is taken as a given value, whereas in the second, (l_s) is taken as a design variable. The optimum design of the floor is found as shown in Tables (2) & (3)

$d_1 = 60 \text{ mm}$

EXAMPLE (3)

In this example the area of tension reinforcement per meter width of slab for the end span is used as a design variable along with the design variables of example (2). Here the same results as those of Table (2) are obtained when C/C beam distance is taken as a given value. On the other hand, when C/C beam distance is taken as a design variable, the results are as shown in Table(4).

EXAMPLE (4)

In order to examine the effect of concrete compressive strength on the cost different grades of concrete are considered in this example. Five design variables are adopted and their optimum values are given in Table (5).

In order to further examine the effect of the compressive strength of concrete on the optimum cost of the floor system, a separate study is carried out. The total depth and tensile reinforcement of the beam are only considered as design variables whereas the given data are:-

$l_s = 3.0 \text{ m}$ $h_f = 150 \text{ mm}$

$Ass_1 = 270 \text{ mm}^2$ $Ass_2 = 270 \text{ mm}^2$

$Ass_3 = 270 \text{ mm}^2$ $Ass_4 = 270 \text{ mm}^2$

Three values of cost ratio R_2 (100, 200, and 1000 N/m^2) are examined. It is

respectively. The effects of these variations are examined and the results are shown in Figs. (5) to (7).

DISCUSSION

Effect of the Design Variables

When the considered design variables are the total depth and tensile reinforcement of the beam, it is found that the formwork cost ratio has a considerable effect on these variables [Table (1)]. The optimum values of the design variables change when the formwork cost ratio is included.

Table (2) gives the optimum thickness of slab, when it is treated as a design variable as well as the design variables of the beam (total depth and tension reinforcement). It has been observed that the thickness and the main tensile reinforcement of the critical sections of slab are independent of the formwork cost ratio. If the slab thickness is not considered as a design variable, the effect of C/C beam distance, as related to the formwork cost ratio, appears in that the program enhances the C/C beam distance without limitation, where the deflection is controlled.

Effect of the Material Properties

It can be seen from Tables (5),(6) and (7) that it would be better in the optimum design of RC structures to use low values of compressive strength of concrete with high yield strength of steel. Table(5) shows that the increase of compressive strength of concrete causes the C/C distance between beams to decrease. This may be due to the fact that the increase in the compressive strength of concrete leads to an increase in the cost of concrete. Therefore, the distance between beams must be reduced to cause a reduction in the size of the slab and the beams. Also the increase in the cost of concrete due to increasing its compressive strength causes the increase of the tension

reinforcement and decrease of the total depth of the beam.

Effect of the Cost Ratios

The effect of the variation in values of concrete and formwork cost ratios on the optimum values of C/C distance between beams, total depth, and tension reinforcement of the beams is shown in Figs.(5),(6) and (7).

The optimum C/C distance between beams (ls) increases with the increase in the relative cost of formwork [Fig.(5)]. This may be attributed to the reduction in the obtained number of beams, which reduces the formwork cost of the floor system. On the other hand, (ls) decreases with the increase in relative cost of concrete. This results from the reduction in the dimensions of the slab and beams, which can be obtained with the decrease in the C/C beam distance (ls).

The depth of beam is found to decrease with increase in the relative cost of concrete as shown in Fig. (6). The same finding for the depth of beam is also noted with the increase in formwork cost to a certain value and then an increase in the depth does occur [Fig.(6)]. The area of tension reinforcement in beams is found to increase with the increase in formwork cost ratio as depicted in Fig. (7). Also the tension reinforcement of the beam decreases with the increase in the concrete cost ratio for high values of formwork cost ratio. This may be attributed to the reduction in the distance between beams which reduces in turn the applied load on the beams.

CONCLUSIONS

Based on the results obtained in this study the following conclusions can be drawn:

- 1) The general program using the interior penalty function method

provides a good means for designing the RC T-beam floor with minimum cost.

2) The concrete and formwork costs are found to have considerable effect on the optimum values for the design variables of the beam, however little effect has been recorded on the optimum values for the design variables of slab.

3) There is no need to take the parameters of slab as design variables when the C/C distance between beams is treated as a given data. The optimum values in this case are determined directly by using the deflection and flexural constraints.

4) From a cost point of view it appears that it is preferable to use low concrete compressive strength in designing RC floor system.

5) The C/C beam distance increases when high values of formwork cost are used.

6) For high values of formwork cost, the minimum cost of the floor system is achieved with the increase in beam tension steel and decrease in beam depth.

7) The C/C distance between beams decreases when high values of concrete cost are used.

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(10)

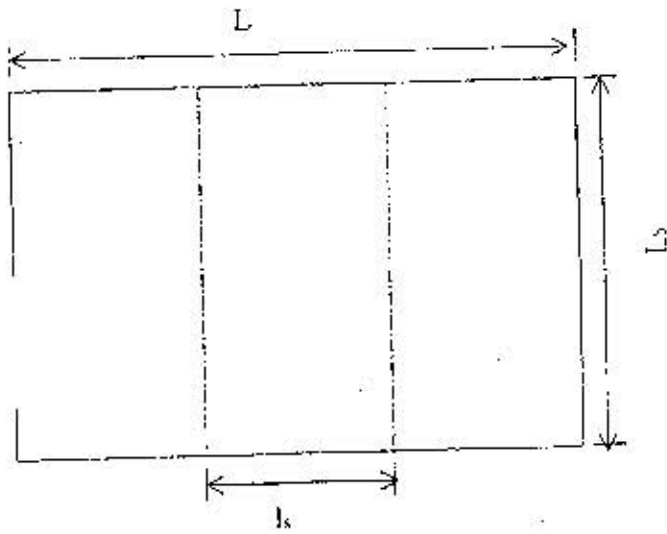


Figure (1): Details of slab and beams arrangement .

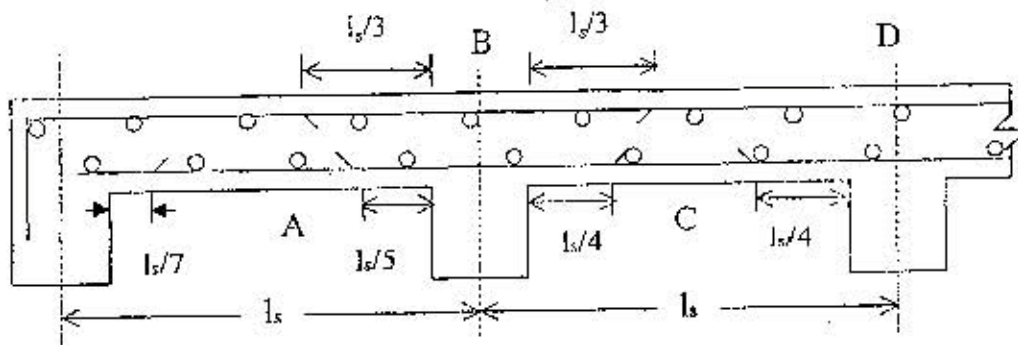


Figure (2): Details of slab reinforcement.

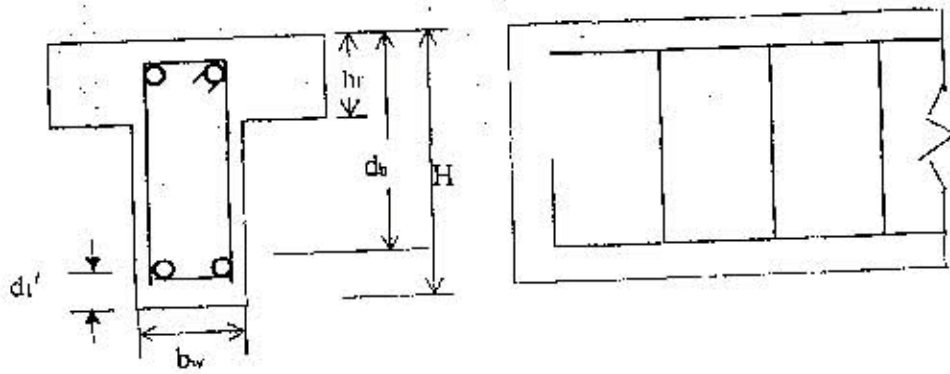


Figure (3): Details of beams reinforcement.

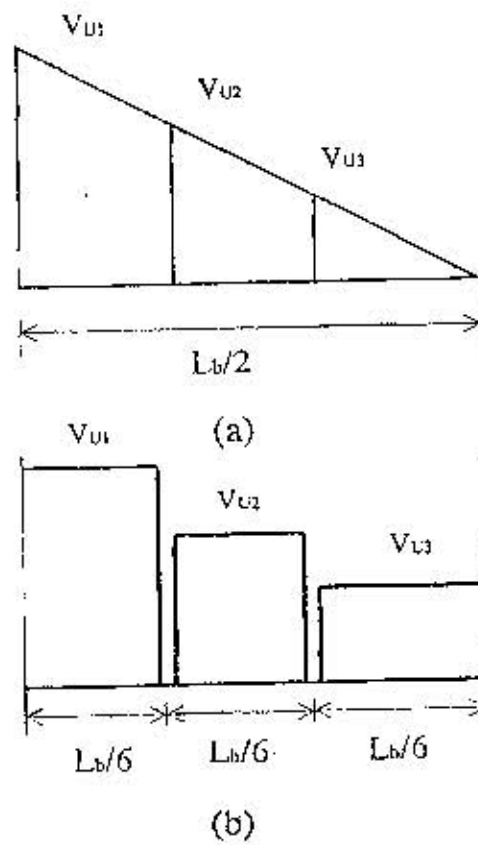


Figure (4): a- Actual shear force diagram for half beam span.
b- Design shear force diagram.

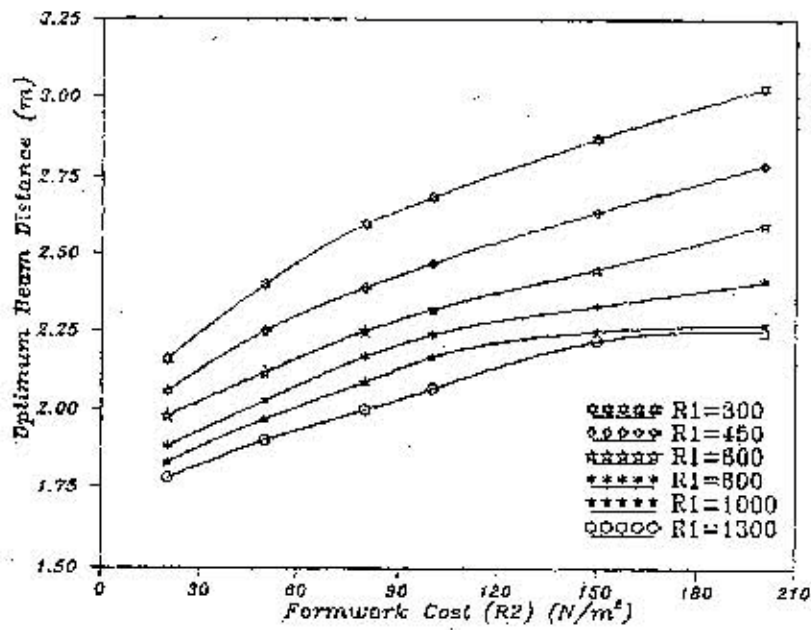


Figure (5): Relationship between optimum c/c beam distance and relative cost of formwork.

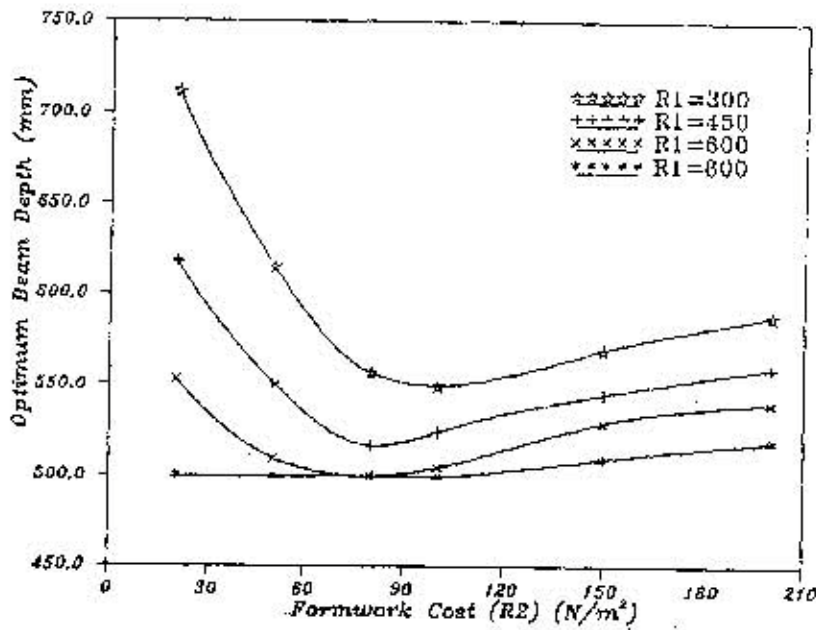


Figure (6): Relationship between optimum depth of the beam distance and relative cost of formwork.

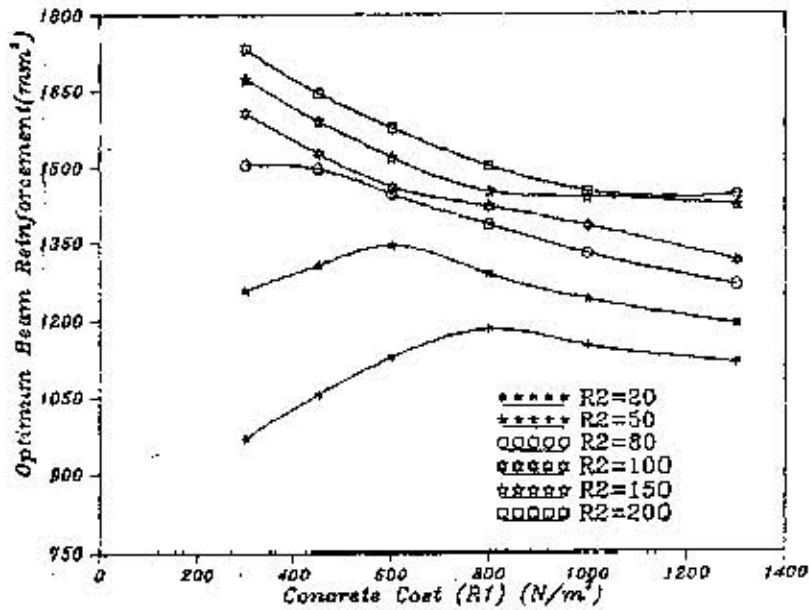


Figure (7): Relationship between optimum beam reinforcement and relative cost of concrete.

Table(1): Results of example (1); $R1=600\text{N/m}^3$ and $R2=80\text{N/m}^2$

| Given data | | | Optimum design variables | | Amount of reinforcement per meter | | | | | Optimum relative cost |
|------------|---------|----------|--------------------------|---------|-----------------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-----------------------|
| N | ls m | hf mm | Asb mm ² | H mm | Asv mm ² | Ass1 mm ² | Ass2 mm ² | Ass3 mm ² | Ass4 mm ² | cost N/m |
| Without R2 | | | | | | | | | | |
| 2 | 4 | 170 | 1515 | 970 | 395 | 385 | 425 | 305 | 385 | 2423 |
| 3 | 3 | 150 | 1300 | 825 | 435 | 270 | 270 | 270 | 270 | 2277* |
| 4 | 2.4 | 150 | 1170 | 745 | 470 | 270 | 270 | 270 | 270 | 2429 |
| With R2 | | | | | | | | | | |
| 2 | 4 | 170 | 2090 | 705 | 550 | 385 | 425 | 305 | 385 | 3610 |
| 3 | 3 | 150 | 1770 | 610 | 590 | 270 | 270 | 270 | 270 | 3527* |
| 4 | 2.4 | 150 | 1595 | 550 | 670 | 270 | 270 | 270 | 270 | 3746 |

*Optimum design

Table(2): Results of example (2); three design variables; $R1=600N/m^2$
and $R2=80N/m^2$

| Given data | | Optimum design variables | | | Amount of reinforcement per meter | | | | | Optimum relative cost |
|------------|---------|--------------------------|------------------------------------|---------|------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-----------------------|
| N | ls m | h _f mm | A _{sb} mm ² | H mm | A _{sv} mm ² | A _{ss1} mm ² | A _{ss2} mm ² | A _{ss3} mm ² | A _{ss4} mm ² | N/m |
| Without R2 | | | | | | | | | | |
| 2 | 4 | 167 | 1505 | 965 | 395 | 390 | 430 | 300 | 390 | 2395 |
| 3 | 3 | 125 | 1265 | 800 | 435 | 280 | 250 | 225 | 250 | 2031 |
| 4 | 2.4 | 100 | 1110 | 705 | 510 | 215 | 200 | 180 | 200 | 1904 |
| 5 | 2 | 85 | 1005 | 635 | 550 | 185 | 165 | 150 | 165 | 1863 |
| 6 | 1.7 | 72 | 930 | 585 | 630 | 160 | 145 | 130 | 145 | 1897 |
| With R2 | | | | | | | | | | |
| 2 | 4 | 167 | 2080 | 700 | 590 | 390 | 430 | 300 | 390 | 3585 |
| 3 | 3 | 125 | 1725 | 590 | 630 | 280 | 250 | 225 | 250 | 3293 |
| 4 | 2.4 | 100 | 1505 | 520 | 710 | 215 | 200 | 180 | 200 | 3230 |
| 5 | 2 | 85 | 1280 | 500 | 710 | 185 | 165 | 150 | 165 | 3265 |

Table(3): Results of example (2); four design variables; $R1=600N/m^2$
and $R2=80N/m^2$

| Optimum design variables | | | | | Amount of reinforcement per meter | | | | | Optimum relative cost |
|--------------------------|---------|----------------------|------------------------------------|---------|------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-----------------------|
| N | ls m | h _f mm | A _{sb} mm ² | H mm | A _{sv} mm ² | A _{ss1} mm ² | A _{ss2} mm ² | A _{ss3} mm ² | A _{ss4} mm ² | N/m |
| Without R2 | | | | | | | | | | |
| 5 | 2.01 | 85 | 1015 | 635 | 550 | 185 | 170 | 150 | 170 | 1872 |
| With R2 | | | | | | | | | | |
| 4 | 2.5 | 105 | 1600 | 510 | 670 | 225 | 210 | 190 | 210 | 3306 |

Table(5): Effect of compressive strength of concrete on optimum design ($R_2=50N/mm^2$).

| Given data | | Optimum design variables | | | | | | Amount of reinforcement per meter | | | | Optimum relative cost |
|-------------------|-------------------|--------------------------|--------------------|-------------|---------------------|--------------|--------------------|-----------------------------------|---------------------|---------------------|--------------------------------|-----------------------|
| f_c N/mm^2 | R_1 N/mm^2 | h_f mm | A_{sp} mm^2 | H mm | A_{ss1} mm^2 | l_s m | A_{sv} mm^2 | A_{ss2} mm^2 | A_{ss3} mm^2 | A_{ss4} mm^2 | Optimum relative cost N/m | |
| 45 | 655 | 87 | 1315 | 500 | 186 | 2.0 | 710 | 170 | 155 | 710 | 2875 | |
| 40 | 576 | 89 | 1340 | 502 | 190 | 2.13 | 710 | 175 | 157 | 710 | 2757 | |
| 35 | 515 | 92 | 1330 | 520 | 195 | 2.19 | 710 | 180 | 162 | 710 | 2644 | |
| 30 | 450 | 94 | 1310 | 545 | 202 | 2.24 | 630 | 185 | 164 | 630 | 2530 | |
| 25 | 400 | 95 | 1280 | 570 | 206 | 2.28 | 630 | 188 | 167 | 630 | 2443 | |
| 20 | 355 | 97 | 1270 | 585 | 210 | 2.32 | 630 | 193 | 172 | 630 | 2383 | |
| 15 | 315 | 100 | 1260 | 610 | 218 | 2.35 | 590 | 200 | 175 | 590 | 2350 | |

Table(4): Results of example (3); five design variables; $R1=600\text{N/m}^3$
and $R2=80\text{N/m}^2$

| Optimum design variables | | | | | Amount of reinforcement per meter | | | | | Optimum relative cost N/m |
|--------------------------|------------|-------------|---------------------------|---------|-----------------------------------|----------------------------|----------------------------|----------------------------|-----|------------------------------|
| N | l_s m | h_f mm | A_{sb} mm^2 | H mm | A_{ss1} mm^2 | A_{ss2} mm^2 | A_{ss3} mm^2 | A_{ss4} mm^2 | | |
| Without R2 | | | | | | | | | | |
| 6 | 1.84 | 77 | 970 | 600 | 170 | 630 | 155 | 135 | 155 | 1865 |
| With R2 | | | | | | | | | | |
| 4 | 2.26 | 95 | 1450 | 500 | 205 | 710 | 190 | 170 | 190 | 3209 |

Table(6): Effect of compressive strength of concrete on the cost of floor system ($R2=200\text{N/m}^2$).

| Given data | | reinforcement | Optimum design variables | | Optimum relative cost N/m |
|---------------------------|------------------------|---------------------------|---------------------------|---------|------------------------------|
| f'_c N/mm^2 | $R1$ N/m^3 | A_{sv} mm^2 | A_{sb} mm^2 | H mm | |
| 45 | 870 | 750 | 2155 | 500 | 5795 |
| 40 | 780 | 750 | 2160 | 500 | 5618 |
| 35 | 695 | 750 | 2165 | 500 | 5448 |
| 30 | 600 | 750 | 2180 | 500 | 5250 |
| 25 | 540 | 670 | 1975 | 545 | 5145 |
| 20 | 475 | 590 | 1775 | 605 | 5035 |
| 15 | 420 | 510 | 1550 | 695 | 4980 |

Table(7): Optimum design of floor system for different values of f_y .

$R_1=450\text{N/m}^3$ and $R_2=50\text{N/m}^2$

| given data | optimum design var. | | dimensions and amount of reinforcement | | | | | | optimum relative cost N/m | |
|--------------------------|---------------------------|--------------------|--|----------------------|---------------------------|----------------------------|----------------------------|----------------------------|------------------------------------|----------------------------|
| | A_{sb} mm^2 | H mm | l_s m | h_c mm | A_{sv} mm^2 | A_{ss1} mm^2 | A_{ss2} mm^2 | A_{ss3} mm^2 | | A_{ss4} mm^2 |
| f_y N/mm^2 | | | | | | | | | | |
| 250 | 2065 | 830 | | | 470 | 378 | 345 | 300 | 345 | 3054 |
| 300 | 1865 | 765 | 0.8 | 150 | 471 | 315 | 300 | 300 | 300 | 2924 |
| 400 | 1590 | 675 | 0.8 | 150 | 550 | 270 | 270 | 270 | 270 | 2759 |
| 450 | 1490 | 640 | | | 550 | 240 | 240 | 240 | 240 | 2655 |